**BIG IDEA** When a set of data points in a situation seems to be showing exponential growth or decay, exponential regression can fit an exponential function to the data points.

After a person takes medicine, the amount of drug left in the person’s body decreases over time. When testing a new drug, a pharmaceutical company develops a mathematical model to quantify this relationship. To find such a model, suppose a dose of 1000 mg of a certain drug is absorbed by a person’s bloodstream. Blood samples are taken every five hours, and the amount of drug remaining in the body is calculated.

Possible data from an experiment are shown in the table and scatterplot below. The scatterplot suggests that an exponential model might be appropriate. Exponential models can be fit to data using methods similar to those that you used to find linear and quadratic models in earlier chapters.

<table>
<thead>
<tr>
<th>Hours Since Drug was Administered</th>
<th>Amount of Drug in Body (mg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1000</td>
</tr>
<tr>
<td>5</td>
<td>550</td>
</tr>
<tr>
<td>10</td>
<td>316</td>
</tr>
<tr>
<td>15</td>
<td>180</td>
</tr>
<tr>
<td>20</td>
<td>85</td>
</tr>
<tr>
<td>25</td>
<td>56</td>
</tr>
<tr>
<td>30</td>
<td>31</td>
</tr>
</tbody>
</table>

**Finding Exponential Models for Data Believed to Be Exponential**

As you know, exponential functions have the form \( y = ab^x \), where \( a \) is the value of \( y \) when \( x = 0 \) and \( b \) is the growth factor during each unit period of time.
Example 1

Find an exponential model to fit the drug absorption data using the initial condition and one other point in the table.

Solution

You need to find \( a \) and \( b \) in the equation \( y = ab^x \). The initial condition occurs when \( x = 0 \), so \( 1000 = ab^0 \).

Since \( b^0 = 1 \), \( a = 1000 \). So the equation is of the form \( y = 1000b^x \).

Choose another point from the table and substitute. We chose (20, 85).

\[
85 = 1000b^{20} \quad \text{Substitute.}
\]

\[
0.085 = b^{20} \quad \text{Divide each side by 1000.}
\]

\[
0.884 \approx b \quad \text{Take the 20th root of each side (or raise each side to the \( \frac{1}{20} \) power).}
\]

So, one model for the data is \( y = 1000 \cdot (0.884)^x \).

Check

Graph this model on a graphing utility. It looks like an exponential decay function with \( y \)-intercept 1000, as you would expect from the data.

You can also use your graphing utility to find an exponential regression model.

Activity

Step 1 Enter the drug absorption data into your calculator. Apply the exponential regression option to find \( a \) and \( b \) and then write an equation to model these data.

Step 2 Your calculator probably gives \( a \) and \( b \) to many digits. The accuracy of the experimental data suggests that a more sensible model rounds \( a \) and \( b \) to three digits each. Rewrite your equation after rounding.

Step 3 Use your regression model from Step 2 and the two-point model \( y = 1000 \cdot (0.884)^x \) from Example 1 to estimate the amount of drug left in a body after 5 hours. Which model better fits the actual 550 mg value from the experiment?

Step 4 Compare the accuracy of the two models by making a table of values for each equation and comparing these values to the actual data. Which model appears to be more accurate overall?
Recall that, by design, regression is intended to fit a model as close as possible to all the data in a set. The Activity shows that a regression model is indeed a better fit to the drug absorption data than a model based on only two points.

**Deciding Whether an Exponential Model Is Appropriate**

For some data you may not be sure that an exponential model is appropriate. In that case, consider two things. First, look at a scatterplot of your data to see if it has the general shape of an exponential function. This is a quick way to check if the growth factor between various data points is relatively constant. Second, find an exponential model, then look at a table of values or the graph of your model to see how well it fits the data.

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**Example 2**

Below are the U.S. box office gross for the first eleven weekends of the release of the movie *Mission Impossible III* in 2006.

<table>
<thead>
<tr>
<th>Weekend</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. Box Office Gross (millions of dollars)</td>
<td>47.70</td>
<td>25.00</td>
<td>11.35</td>
<td>7.00</td>
<td>4.68</td>
<td>3.02</td>
<td>1.34</td>
<td>0.72</td>
<td>0.49</td>
<td>0.31</td>
<td>0.20</td>
</tr>
</tbody>
</table>

a. Fit an exponential model for box office gross based on the number of weekends the movie has been out and determine if the exponential model is appropriate.

b. Use the model to estimate the movie’s gross in its 12th weekend.

**Solution**

a. The scatterplot appears to have an exponential shape so an exponential model seems appropriate. Perform exponential regression to obtain

\[ y = 69.617842 \cdot (0.57873986)^x. \]

Round \( a \) and \( b \) based on the actual data's accuracy.

\[ y = 69.6 \cdot 0.579^x \]

When it is added to the scatterplot, the graph of the model closely follows the pattern of the data points. It appears to be a good model. Exponential decay models the data closely, but could a parabola fit the data better?
Graph the data together with the graph of a quadratic regression equation.
Exponential decay is a better fit. This makes sense because you know that movie gross earnings typically continue to decrease each successive week after release. Gross earnings do not reach a minimum and then continue to climb indefinitely, so a quadratic model is illogical.

b. Substitute \( x = 12 \) in for \( x \) in the model.
\[
y = 69.6 \times 0.579^{12}
\]
\[
y \approx 0.099
\]
According to this model, the movie grossed about $99,000 in its 12th week.

The weekend box office gross earnings of many movies decline according to an exponential pattern. This helps theater managers estimate how long they should continue to show a movie.

Questions

**COVERING THE IDEAS**

In 1 and 2, refer to the drug absorption data given at the beginning of the lesson.

1. a. Use the point (10, 316) and the method of Example 1 to find an equation to model the data.
   b. Use your equation from Part a to predict when there will be less than 10 mg of the drug left in the patient’s body.

2. Use the model developed in the Activity to predict when the amount of the drug in the patient’s body will fall below 10 mg.

3. Can you use the method of Example 1 to find an equation to model the data in Example 2? Why or why not?

4. **Multiple Choice** Which scatterplot(s) below show(s) data that could be reasonably represented by an exponential model?

   A
   ![Graph A]
   
   B
   ![Graph B]
   
   C
   ![Graph C]
5. **Recall the chart of cell phone subscriptions from the opening of this chapter. The table at the right presents the data for seven years.**

   **a.** Find an exponential model for the growth of cell phone subscriptions.

   **b.** Compare cell phone growth and population growth. The population of the United States was 301.1 million in 2007 and growing exponentially at 0.89% per year. In which year does your model first predict that there will be at least one cell phone subscription per person?

### APPLYING THE MATHEMATICS

6. **The gross earnings for the first three weekends of a popular movie are given in the table at the right.**

   **a.** Find an exponential model for the decline in weekend gross.

   **b.** Studio executives want to pull the movie from theaters before its weekend gross drops below $1 million. How many weekends should the studio expect to keep the movie in theaters?

7. **Deven dropped 3728 pennies on the floor. He picked up all the coins showing heads and set them aside. Then he counted the tails, put them in a container, mixed them up, and dropped them. He repeated this several times and made a table of the results as shown at the right. Unfortunately, he forgot to record a couple of entries.**

   **a.** Create an exponential model of the data using the 5 completed row entries.

   **b.** Use the model to estimate in the missing data.

   **c.** Deven’s brother Tyler said the model looked like a half-life model for radioactive elements. Explain whether Tyler is correct.

8. **The table at the right contains the cumulative monthly English-language article total for the website Wikipedia.** The timetable ranges from March, 2005 to February, 2006.

   **a.** Develop an exponential model for the number \( y \) of English-language articles in Wikipedia in month \( x \). Does this model seem reasonable based on the graph?

   **b.** Develop a quadratic model for the number \( y \) of English-language articles in Wikipedia in month \( x \). Does this model seem reasonable based on the graph?

   **c.** Develop a linear model for the number \( y \) of English-language articles in Wikipedia in month \( x \). Does this model seem reasonable based on the graph?

   **d.** Use each model to predict how many articles will be on Wikipedia in the current year. Check your results on the Internet to determine the most accurate model.
9. a. Graph \( y = e^{5x} \) for \(-2 < x < 2\). Label the coordinates of three points.
b. State the domain and range of this function. (Lesson 9-3)

10. Rumors and fads spread through a population in a process known as social diffusion. Social diffusion can be modeled by \( N = Ce^{kt} \), where \( N \) is the number of people who have heard the rumor after \( t \) days. Suppose four friends start a rumor and two weeks later 136,150 people have heard the rumor. (Lesson 9-3)
   a. In this situation, what is the value of \( C \)?
b. What is the value of \( k \)?
c. Graph the growth of the rumor during the first two weeks.
d. How many people heard the rumor after 10 days?
e. How long will it take for one million people to have heard the rumor?

11. Anne put $4800 in an account with a 3.125% annual interest rate. What will be her balance if she leaves the money untouched for four years compounded
   a. annually?
   b. daily?
c. continuously? (Lessons 9-3, 7-4)

12. Consider the sequence defined by
   \[
   \begin{align*}
   h_1 &= 8.375 \\
   h_n &= 0.8h_{n-1}, \text{ for integers } n \geq 2
   \end{align*}
   \] (Lessons 9-2, 9-1, 7-5, 3-1)
   a. List the first five terms of this sequence.
b. Which phrase best describes this sequence:
   exponential growth, exponential decay, constant increase, or constant decrease?

In 13 and 14, simplify without using a calculator. (Lessons 7-8, 7-7, 7-3)

13. \( \left( \frac{1}{2} \right)^{-4} \)
14. \( 32^{\frac{5}{6}} \)

15. Simplify \( (x + y)^2 - (x - y)^2 \). (Lesson 6-1)

16. Suppose \( t \) varies inversely with \( r \), and \( t = 24 \) when \( r = 24 \). Find \( t \) when \( r = 6 \). (Lesson 2-2)

EXPLORATION

17. Find weekend box office gross data for a movie that has only been out three or four weeks. Develop a model for its decline in gross and predict the gross for future weekends. Track your predictions to see how accurate your model is. (You may want to update your model as new data comes in.) What factors may cause a movie’s weekend gross to not follow an exponential model?